

Complex-mass scheme and resonances in EFT

T. Bauer*, D. Djukanovic[†], J. Gegelia^{*,**}, S. Scherer* and L. Tiator*

**Institut für Kernphysik, Johannes Gutenberg-Universität, D-55099 Mainz, Germany*

[†]Helmholtz Institut Mainz, D-55099 Mainz, Germany

***High Energy Physics Institute of TSU, 0186 Tbilisi, Georgia*

Abstract.

The complex-mass scheme (CMS) provides a consistent framework for dealing with unstable particles in quantum field theory and has been successfully applied to various loop calculations. As applications of the CMS in chiral effective field theory we consider the form factor of the pion in the time-like region and the magnetic moment of the Roper resonance.

Keywords: EFT, Complex-mass scheme, pion form factors, magnetic moment, Roper resonance

PACS: 12.39.Fe., 11.10.Gh, 03.70.+k

INTRODUCTION

Chiral perturbation theory (ChPT) is an established low energy effective field theory of QCD in the vacuum sector [1, 2]. While the problem of including the nucleon and the Δ in this approach has been resolved [3], the treatment of other states is more complicated. A solution to the problem of inclusion of unstable particles in chiral EFT is provided by the complex-mass scheme (CMS) [4, 5]. As an application of CMS the mass and the width of the ρ meson and the Roper resonance have been considered [6, 7]. In this contribution we present the results for the form factor of the pion in the time-like region up to $q^2 \sim 1 \text{ GeV}^2$ and the magnetic moment of the Roper resonance up to $\mathcal{O}(q^3)$.

PION FORM FACTOR

We start with the effective Lagrangian of pions and ρ mesons given as [6]

$$\begin{aligned} \mathcal{L} = & \frac{F^2}{4} \text{Tr} [D_\mu U (D^\mu U)^\dagger] + \frac{F^2}{4} \text{Tr} [\chi U^\dagger + U \chi^\dagger] \\ & + \frac{M^2 + c_x \text{Tr} [\chi_+]/4}{g^2} \text{Tr} [(g\rho^\mu - i\Gamma^\mu)(g\rho_\mu - i\Gamma_\mu)] \\ & - \frac{1}{2} \text{Tr} [\rho_{\mu\nu} \rho^{\mu\nu}] + i d_x \text{Tr} [\rho^{\mu\nu} \Gamma_{\mu\nu}] - \frac{\sqrt{2}}{2} f_V \text{Tr} \{\rho_{\mu\nu} f^{\mu\nu}\} + \dots, \end{aligned} \quad (1)$$

where dots stand for terms with more derivatives/fields and

$$\begin{aligned} U(x) &= u^2(x) = \exp \left[\frac{i\Phi(x)}{F} \right], \quad D_\mu A = \partial_\mu A - i v_\mu A + i A v_\mu, \\ \chi_+ &= M^2 (U^\dagger + U), \quad \rho_\mu = \rho_\mu^a \frac{\tau^a}{2}, \quad \Gamma_{\mu\nu} = \partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu + [\Gamma_\mu, \Gamma_\nu], \end{aligned}$$

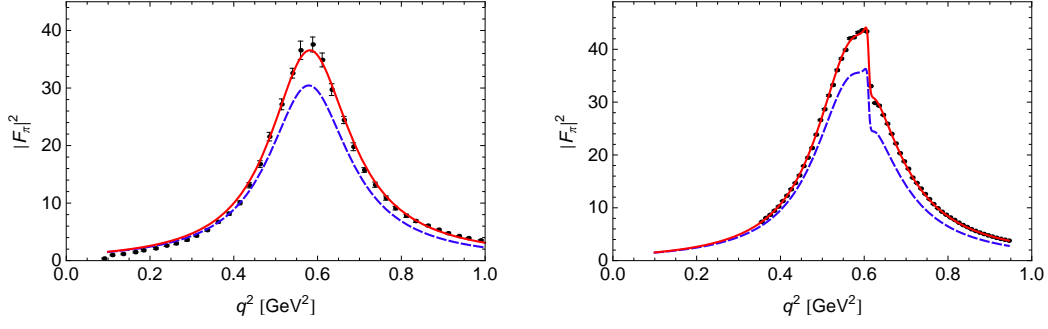


FIGURE 1. Electromagnetic form factor of the pion. Left panel - extracted from $\tau^- \rightarrow \nu_\tau \pi^- \pi^0$ decay, data - from Ref. [11]. Right panel - extracted from $e^+ e^- \rightarrow \pi^+ \pi^-$, data - from Ref. [12]. The dashed lines show the results of the tree diagrams and the solid lines of the tree plus one-loop diagrams respectively.

$$\begin{aligned}
 \Gamma_\mu &= \frac{1}{2} \left[u^\dagger \partial_\mu u + u \partial_\mu u^\dagger - i \left(u^\dagger v_\mu u + u v_\mu u^\dagger \right) \right], \\
 \rho^{\mu\nu} &= \partial^\mu \rho^\nu - \partial^\nu \rho^\mu - ig [\rho^\mu, \rho^\nu], \\
 f_{\mu\nu} &= u(\partial_\mu v_\nu - \partial_\nu v_\mu) u^\dagger + u^\dagger (\partial_\mu v_\nu - \partial_\nu v_\mu) u.
 \end{aligned} \tag{2}$$

Here F is the pion-decay constant in the chiral limit, M is the pion mass at leading-order, v_μ and a_μ are the external vector and axial vector fields and $M^2 = 2g^2 F^2$ [8, 9, 10].

In the following we use the CMS as renormalization scheme. To determine the chiral order of a diagram we consider all possible flows of the external momenta through the internal lines and determine the chiral order for each flow: Pion propagators which do not carry large momenta are counted as $\mathcal{O}(q^{-2})$, and those carrying large momenta - as $\mathcal{O}(q^0)$, vector meson propagators not carrying large momenta - as $\mathcal{O}(q^0)$, and those carrying large momenta - as $\mathcal{O}(q^{-1})$. The pion mass counts as $\mathcal{O}(q^1)$, the vector meson mass - as $\mathcal{O}(q^0)$ and the width - as $\mathcal{O}(q^1)$. Vertices generated by $\mathcal{L}_\pi^{(n)}$ count as $\mathcal{O}(q^n)$. On the other hand derivatives acting on heavy vector mesons are counted as $\mathcal{O}(q^0)$. The chiral order of the diagram is given by the smallest amongst the orders of different flows.

We have calculated tree and one-loop order contributions to the pion form factor $F(q^2)$. After renormalization $F(q^2)$ is dominated by tree order diagrams with vector meson exchange. We fitted the available parameters of the effective Lagrangian to the pion form factor extracted from the $\tau^- \rightarrow \nu_\tau \pi^- \pi^0$ decay. We also calculated the form factor at tree order for the same values of the parameters by switching off the loop contributions. Our results are plotted in the left panel of Fig. 1.

The $\rho^0 - \omega - A$ mixing plays an important role in describing the pion form factor extracted from $e^+ e^- \rightarrow \pi^+ \pi^-$. Within the QFT formalism the above mixing is taken into account by solving the system of coupled equations for dressed propagators. We take into account the $\rho - \omega - A$ mixing only at tree order. However we allow the mixing parameters to become complex. By fitting the mixing parameters to the data we obtain the results for the pion form factor plotted in the right panel of Fig. 1 together with the experimental data and the form factor at tree order for the same values of the parameters.

MAGNETIC MOMENT OF THE ROPER RESONANCE

The leading-order effective Lagrangian relevant for the calculation of the magnetic moment at $\mathcal{O}(q^3)$ is given by

$$\begin{aligned}\mathcal{L}_0 &= \bar{N}(i\not{D}-m_N)N + \bar{R}(i\not{D}-m_R)R \\ &- \bar{\Psi}_\mu \xi^{\frac{3}{2}} \left[(i\not{D}-m_\Delta) g^{\mu\nu} - i(\gamma^\mu D^\nu + \gamma^\nu D^\mu) + i\gamma^\mu \not{D} \gamma^\nu + m_\Delta \gamma^\mu \gamma^\nu \right] \xi^{\frac{3}{2}} \Psi_\nu.\end{aligned}\quad (3)$$

Here, N and R denote nucleon and Roper resonance fields and Ψ_ν is the Rarita-Schwinger field of the Δ resonance. The covariant derivatives are defined as

$$\begin{aligned}D_\mu H &= \left(\partial_\mu + \Gamma_\mu - i v_\mu^{(s)} \right) H, \\ (D_\mu \Psi)_{\nu,i} &= \partial_\mu \Psi_{\nu,i} - 2i \varepsilon_{ijk} \Gamma_{\mu,k} \Psi_{\nu,j} + \Gamma_\mu \Psi_{\nu,i} - i v_\mu^{(s)} \Psi_{\nu,i},\end{aligned}\quad (4)$$

where H stands either for the nucleon or the Roper resonance and $u = \sqrt{U}$.

Further interaction terms \mathcal{L}_R , \mathcal{L}_{NR} , and $\mathcal{L}_{\Delta R}$ are constructed in analogy to Ref. [13]:

$$\begin{aligned}\mathcal{L}_R^{(1)} &= \frac{g_R}{2} \bar{R} \gamma^\mu \gamma_5 u_\mu R, \quad \mathcal{L}_R^{(2)} = \bar{R} \left[\frac{c_6^*}{2} f_{\mu\nu}^+ + \frac{c_7^*}{2} v_{\mu\nu}^{(s)} \right] \sigma^{\mu\nu} R + \dots, \\ \mathcal{L}_R^{(3)} &= \frac{i}{2} d_6^* \bar{R} \left[D^\mu, f_{\mu\nu}^+ \right] D^\nu R + \text{h.c.} + 2i d_7^* \bar{R} \left(\partial^\mu v_{\mu\nu}^{(s)} \right) D^\nu R + \text{h.c.} + \dots, \\ \mathcal{L}_{NR}^{(1)} &= \frac{g_{NR}}{2} \bar{R} \gamma^\mu \gamma_5 u_\mu N + \text{h.c.}, \quad \mathcal{L}_{\Delta R}^{(1)} = -g_{\Delta R} \bar{\Psi}_\mu \xi^{\frac{3}{2}} (g^{\mu\nu} + \tilde{z} \gamma^\mu \gamma^\nu) u_\nu R + \text{h.c.}, \\ u_\mu &= i \left[u^\dagger \partial_\mu u - u \partial_\mu u^\dagger - i \left(u^\dagger v_\mu u - u v_\mu u^\dagger \right) \right], \\ v_{\mu\nu}^{(s)} &= \partial_\mu v_\nu^{(s)} - \partial_\nu v_\mu^{(s)}, \quad f_{\mu\nu}^+ = u f_{\mu\nu} u^\dagger + u^\dagger f_{\mu\nu} u, \quad f_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu]\end{aligned}\quad (5)$$

and g_R , g_{NR} , $g_{\Delta R}$, c_6^* , c_7^* , d_6^* , d_7^* are unknown coupling constants and we take $\tilde{z} = -1$.

We renormalize loop diagrams by applying the CMS and use the standard power counting of Ref. [14]. Following Ref. [15], we parameterize the vertex function as

$$\sqrt{Z_R} \bar{w}^i(p_f) \Gamma^\mu(p_f, p_i) w^j(p_i) \sqrt{Z_R} = \bar{w}^i(p_f) \left[\gamma^\mu F_1(q^2) + \frac{i \sigma^{\mu\nu} q_\nu}{2m_N} F_2(q^2) \right] w^j(p_i), \quad (6)$$

where $q = p_f - p_i$, m_N is the mass of the nucleon, and \bar{w}^i and w^j denote "Dirac spinors" with complex masses z . To $\mathcal{O}(q^3)$ the vertex function $\Gamma^\mu(p_f, p_i)$ obtains contributions from three tree diagrams and fourteen loop diagrams. We obtain $F_1(0) = (1 + \tau_3)/2$ and the power-counting-violating loop contributions to the magnetic form factor are absorbed in the couplings c_6^* and c_7^* . The tree order result for $\kappa_R = F_2(0)$ is given by

$$\kappa_R^{\text{tree}} = 2m_N \left(\frac{c_7^*}{2} + \tau_3 c_6^* \right). \quad (7)$$

The estimated value of the loop contributions to the anomalous magnetic moment is

$$\kappa_R = (0.055 + 0.090i) - (0.223 + 0.156i) \tau_3 \quad (8)$$

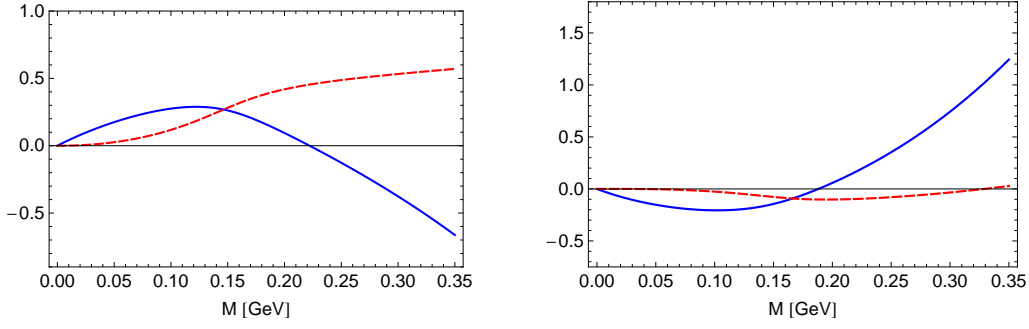


FIGURE 2. One-loop contributions to the anomalous magnetic moment of the neutral(left) and the charged(right) Roper resonances as functions of the pion mass M . The solid and dashed lines indicate the real and imaginary parts, respectively.

and Fig. 2 shows the loop contribution as a function of the lowest-order pion mass M .

SUMMARY

We have presented the results for the form factor of the pion and the magnetic moment of the Roper resonance in the framework of chiral EFT with resonances. Taking into account the $\rho^0 - \omega - A$ mixing at tree level and by fitting the parameters of the Lagrangian a satisfactory description has been obtained for form factors obtained both from $\tau^- \rightarrow \nu_\tau \pi^- \pi^0$ and $e^+ e^- \rightarrow \pi^+ \pi^-$. While no measured value of the magnetic moment of the Roper resonance is available, our expressions could be used in lattice extrapolations.

ACKNOWLEDGMENTS

This work was supported by the Deutsche Forschungsgemeinschaft (SFB 443).

REFERENCES

1. S. Weinberg, *Physica A* **96**, 327 (1979).
2. J. Gasser and H. Leutwyler, *Ann. Phys. (N.Y.)* **158**, 142 (1984).
3. S. Scherer, *Prog. Part. Nucl. Phys.* **64**, 1 (2010).
4. R. G. Stuart, in *Z⁰ Physics*, ed. J. Tran Thanh Van (Editions Frontieres, Gif-sur-Yvette, 1990), p.41.
5. A. Denner, S. Dittmaier, M. Roth and D. Wackeroth, *Nucl. Phys. B* **560**, 33 (1999).
6. D. Djukanovic, J. Gegelia, A. Keller and S. Scherer, *Phys. Lett. B* **680**, 235 (2009).
7. D. Djukanovic, J. Gegelia and S. Scherer, *Phys. Lett. B* **690**, 123 (2010).
8. K. Kawarabayashi and M. Suzuki, *Phys. Rev. Lett.* **16**, 255 (1966).
9. Riazuddin and Fayyazuddin, *Phys. Rev.* **147**, 1071 (1966).
10. D. Djukanovic, M.R.Schindler, J.Gegelia, G.Japaridze, S.Scherer, *Phys. Rev. Lett.* **93**, 122002(2004).
11. S. Schael *et al.* [ALEPH Collaboration], *Phys. Rept.* **421**, 191 (2005).
12. F. Ambrosino *et al.* [KLOE Collaboration], *Phys. Lett. B* **670**, 285 (2009).
13. B. Borasoy, P. C. Bruns, U.-G. Meißner, and R. Lewis, *Phys. Lett. B* **641**, 294 (2006).
14. S. Weinberg, *Nucl. Phys. B* **363**, 3 (1991).
15. J. Gegelia and S. Scherer, *Eur. Phys. J. A* **44**, 425 (2010).